

# UV-COMPLETING GHOST INFLATION

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## Abstract

We present a setup that provides a UV-completion of the ghost inflation model up to a scale which can be almost as high as the Planck mass. This is achieved by coupling the inflaton to the Lorentz-violating sector described by the Einstein-aether theory or its khronometric version. Compared to previous works on ghost inflation our setup allows to go beyond the study of small perturbations and include the background dynamics in a unified framework. In the specific regime when the expansion of the Universe is dominated by the kinetic energy of the inflaton we find that the model predicts rather high tensor-to-scalar ratio  $r \sim 0.1$  and non-Gaussianity of equilateral type with  $f_{NL} \sim -40$ .

## 1 Introduction

All structures in the observed Universe, such as galaxy clusters, galaxies and stars are believed to arise from tiny quantum fluctuations, amplified during a primordial stage in the expansion of the Universe. The most successful model of this stage is the inflationary theory

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which fits very well all current cosmological data. Generally, inflation is supposed to occur at very high energies (probably a few orders below the Planck mass), where the effects of new unknown physics may show up. Thus the inflationary period provides a unique opportunity to probe these scales through cosmological observations. An intriguing possibility is that the known space-time symmetries become invalid at the inflationary energies. One of such symmetries is Lorentz invariance (LI) which is at the basis of the highly successful Standard Model of particle physics and General Relativity (GR). However, beautiful as it is, GR suffers from the problem of non-renormalizability precluding it from being a consistent quantum theory at energies above the Planck scale. It has been suggested by Hořava [1] that the situation can be improved by allowing LI to be violated at high energy. A consistent implementation of this proposal [2] involves additional light degrees of freedom in the gravitational sector that introduce departures from LI even at energies well below Planckian. The description at such energies is provided by the so-called “khronometric” model [3], which can be considered as a variant of the phenomenological “Einstein-aether” theory [4, 5] for the study of Lorentz-violating (LV) effects in gravity (see [6, 7] for the precise relationship between the two models). The existing astrophysical and cosmological data significantly constrain the parameters of the model [8, 3, 9, 10, 11, 12, 13], but still leave open a theoretically motivated portion of the parameter space. It is worth mentioning that any application of these ideas to realistic model building must include a mechanism that would prevent significant percolation of LI breaking from gravity into the Standard Model sector where LI has been tested with an outstanding precision; several options have been discussed in [14, 15, 16, 17].

LV in gravity, though tightly constrained at low energies accessible to current experiments, may be significantly stronger during inflation. An interesting alternative to the standard slow-roll inflation involving LV in gravity is provided by the ghost inflation model [18] and its “tilted” extension [19]. The important feature of these models is the modified dispersion relation for the excitations of the inflaton<sup>1</sup>. The dispersion relation is quadratic,  $\omega^2 \propto k^4$ , in the case of the original ghost inflation and linear,  $\omega^2 = \delta^2 \cdot k^2$ , with the small “sound speed”  $\delta \ll 1$  in the tilted version. This, combined with the specific form of the interactions, leads to interesting predictions for the amplitude and shape of non-Gaussianity, which have been constrained by the Planck results [20].

The ghost inflation models are formulated as effective field theories (EFT) for cosmological perturbations valid below a certain cutoff. While this approach has the advantage of

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<sup>1</sup>By inflaton we loosely understand the field generating the primordial perturbations.

being very general, it is unable to fully capture the evolution of the inflaton background; in particular, it is problematic to incorporate in it the end of inflation and reheating. Besides, if one assumes that the ghost condensate present at inflation persists till today, stability requires the cutoff to be rather low [21, 22]. This makes a UV-completion of ghost inflation desirable.

In this paper we show that ghost inflation can be embedded in the framework of the khronometric or Einstein-aether gravity. The latter being also an EFT, it has a cutoff as well. But this can be as high as just a few orders of magnitude below the Planck mass. In other words, khronometric / Einstein-aether can provide a UV-completion for the ghost condensate model almost up to the Planck scale. This allows to describe the evolution of the background and perturbations within a self-contained theory. Interestingly, the UV-completion happens without restoration of the broken Lorentz symmetry, similar to the case recently discussed in [23]. Further, in the khronometric version of the setup there is a potential UV-completion all the way above the Planck scale in the form of Hořava gravity.

We study the observational signatures of the UV-completed model. For two reasons the analysis somewhat differs from the discussion of the generic ghost inflation present in the literature. First, the ability to keep the background evolution under control allows to consider the situation when the VEV of the inflaton time derivative  $\langle \dot{\Theta} \rangle$  — the “ghost condensate” — varies with time. This produces a contribution into the tilt of the power spectrum in addition to that coming from a potential for  $\Theta$ , which in some cases can actually be dominant. Second, the ghost inflation is characterized essentially by three energy scales: the scale  $\rho_{inf}^{1/4}$  of the inflationary energy density; the scale  $M$  of the ghost condensate  $M^2 = \langle \dot{\Theta} \rangle$  which determines the strength of the inflaton self-interaction producing the leading non-Gaussianity; and the scale  $M'$  suppressing terms with higher spatial derivatives in the quadratic effective action for inflaton perturbations. In the previous treatments of the ghost inflation the latter two scales have been commonly assumed to be of the same order with the first scale being much higher,  $\rho_{inf}^{1/4} \gg M \sim M'$ . We will see that in the UV-completed model where all the above scales are derived quantities,  $M$  and  $M'$  have different parameter dependence, and the requirement that the EFT description extends to the inflationary background imposes the hierarchy<sup>2</sup>  $M \gg M'$ . On the other hand,  $\rho_{inf}^{1/4}$  can naturally be of the same order as  $M$ . As a result of this new hierarchy, the amplitude of non-Gaussianity is somewhat suppressed

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<sup>2</sup>At first sight, this hierarchy might seem surprising from the viewpoint of the EFT for the inflaton *perturbations*. However, it is straightforward to verify that it is stable under radiative corrections and thus perfectly natural.

compared to the original prediction of the ghost inflation; still, it remains large enough to be observationally interesting.

The paper is organized as follows. In Sec. 2 we describe the model and identify the inflationary regime that reproduces ghost inflation. In Sec. 3 we study linear cosmological perturbations emphasizing the similarities and differences with the generic ghost inflation treatment. Sec. 4 contains calculation of the bispectrum. In Sec. 5 we apply our results to the special case when the background dynamics is dominated by the kinetic energy of the inflaton and derive observational constraints on the model parameters in this case. Sec. 6 is devoted to conclusions. Some details of the analysis are postponed to the Appendices.

## 2 Fast-roll inflation

We start with the class of gravity theories containing in addition to the metric  $g_{\mu\nu}$  a dynamical time-like vector field  $u_\mu$  with unit norm,<sup>3</sup>

$$u_\mu u^\mu = 1. \quad (1)$$

This field, called aether, exists in every point of space-time and defines a preferred reference system, breaking the local LI of GR down to the subgroup of spatial rotations around this vector. The dynamics of  $u_\mu$  is described by the most general covariant action containing up to two derivatives,

$$S_{[EH]} + S_{[u]} = -\frac{M_0^2}{2} \int d^4x \sqrt{-g} (R + K^{\mu\nu}{}_{\rho\sigma} \nabla_\mu u^\rho \nabla_\nu u^\sigma), \quad (2)$$

where

$$K^{\mu\nu}{}_{\rho\sigma} = c_1 g^{\mu\nu} g_{\rho\sigma} + c_2 \delta_\rho^\mu \delta_\sigma^\nu + c_3 \delta_\sigma^\mu \delta_\rho^\nu + c_4 u^\mu u^\nu g_{\rho\sigma}, \quad (3)$$

$c_1, \dots, c_4$  are dimensionless parameters and  $M_0$  is related to the Planck mass,

$$M_P^2 = M_0^2 \left( 1 - \frac{c_1 + c_4}{2} \right). \quad (4)$$

This class of theories has been introduced in [4] (see also [5]) and received the name of Einstein-aether models.

It is possible to impose further restriction on  $u_\mu$  to make it hypersurface-orthogonal. This condition can be solved explicitly in terms of a scalar field

$$u_\mu \equiv \frac{\nabla_\mu \sigma}{\sqrt{\nabla^\nu \sigma \nabla_\nu \sigma}}, \quad (5)$$

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<sup>3</sup>Our signature convention is  $(+, -, -, -)$ .

where  $\sigma(t, \mathbf{x})$  is assumed to have a non-vanishing time-like gradient everywhere. Then out of the four terms with the derivatives of  $u_\mu$  in (2) only three are independent and the theory is parameterized by the following combinations,

$$\alpha \equiv c_1 + c_4, \quad \beta \equiv c_1 + c_3, \quad \lambda \equiv c_2. \quad (6)$$

The geometrical meaning of  $\sigma$  is that it labels the slices of a preferred space-time foliation. From the physical viewpoint  $\sigma$  sets a preferred time variable: hence the name “khronon”. The theory with the action (2) and  $u_\mu$  expressed as in (5) was introduced in [3] under the name “khronometric” model. Note that it is invariant under reparameterizations of  $\sigma$ ,

$$\sigma \mapsto \tilde{\sigma}(\sigma), \quad (7)$$

where  $\tilde{\sigma}(\sigma)$  is an arbitrary monotonic function. It has been shown to arise as the low-energy limit of Hořava gravity [1]. In other words, Hořava gravity can potentially provide a UV-completion to the model at trans-Planckian scales.

The generic Einstein-aether theory differs from the khronometric version by the presence of vector modes, while the scalar and tensor sectors in the two models are identical.<sup>4</sup> We will see in the next section that vector perturbations are not generated during inflation for the choice of parameters we are interested in. Therefore, without loss of generality, we will focus on the khronometric case and use the parameterization (6). We will assume throughout the paper that the parameters  $\alpha, \beta, \lambda$  are of the same order and use  $\alpha$  in various estimates as a shorthand for the whole set  $\alpha, \beta, \lambda$ . Then the khronometric gravity is a valid EFT up to the scale<sup>5</sup> [2, 3],

$$\Lambda_{\text{cutoff}} = M_0 \sqrt{\alpha}, \quad (8)$$

which is only a few orders below the Planck mass if  $\alpha$  is not extremely small. To avoid instabilities and negative energy the parameters must satisfy [25, 3]

$$0 < \alpha < 2, \quad 0 < \beta + \lambda.$$

Observations require  $\alpha, \beta, \lambda$  to be small. In particular, in the absence of fine-tuning between  $\alpha, \beta, \lambda$  the Solar System constraints [27] on the post-Newtonian parameters  $\alpha_1^{PPN}, \alpha_2^{PPN}$  describing LV can be translated into the bounds,

$$|\alpha|, |\beta|, |\lambda| \lesssim 10^{-7}. \quad (9)$$

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<sup>4</sup>Another difference is the instantaneous interaction arising in the khronometric model [24]. However, this is irrelevant for the local physics studied in this paper.

<sup>5</sup>A nice study of this subject for the generic Einstein-aether theory is given in [26].

Even stronger constraint on  $\hat{\alpha}_2^{PPN}$  — the analog of  $\alpha_2^{PPN}$  in strong gravitational field — was recently obtained from pulsar timing [11]. However, we will disregard these constraints in this paper for two reasons. First, a single fine-tuning  $\alpha = 2\beta$  is sufficient to make both PPN parameters vanish [3].<sup>6</sup> Then one is left with much weaker bounds on  $\alpha, \beta, \lambda$  at the level of per cent from emission of the gravitational waves [9, 12, 13] and late-time cosmology [10]. Second, the values of  $\alpha, \beta, \lambda$  during inflation can be different from the present epoch (e.g., they can depend on the inflaton field) and may well exceed (9). Thus, the only a priori assumption we are going to make to simplify the calculations is  $\alpha, \beta, \lambda \ll 1$ . This will be validated at the end by the constraints on the primordial spectrum following from the Planck data.

We now introduce the inflaton field  $\Theta$ . As it is well-known, to sustain inflation the potential for  $\Theta$  must be very flat which can be achieved by imposing an approximate shift symmetry,

$$\Theta \mapsto \Theta + \text{const} . \quad (10)$$

Assume for a moment that this symmetry is exact. Then only derivatives of  $\Theta$  can appear in the action. Allowing for the coupling between  $\Theta$  and the aether we obtain the most general action with up to 2 derivatives,

$$S_{[\Theta]} = \int d^4x \sqrt{-g} \left[ \frac{1}{2} g^{\mu\nu} \partial_\mu \Theta \partial_\nu \Theta + \frac{\varkappa}{2} (u^\mu \partial_\mu \Theta)^2 - \mu^2 u^\nu \partial_\nu \Theta - V \right] , \quad (11)$$

where  $\varkappa, \mu$  and  $V$  are constants. One will easily convince oneself that the cutoff of this action combined with (2) is still given by (8). Note that it is technically natural for the mass parameter  $\mu$  to be smaller than  $\Lambda_{\text{cutoff}}$  as it is protected from large quantum corrections by the discrete symmetry  $\Theta \mapsto -\Theta$  obeyed by the rest of the action. The model with non-minimal coupling between the aether and the inflaton similar to (11) was first introduced in [28] and has been recently studied in [29]. In [30, 10] the model with  $V = 0$  was suggested as a possible Lagrangian for dark energy naturally protected from large quantum corrections. To avoid confusion, we point out that the model of this paper differs from that considered in [31] where the khronon itself plays the role of the inflaton.

Let us analyze the background cosmology in the constructed model. Assuming the metric

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<sup>6</sup>This fine-tuning also greatly suppresses the strong-field PPN parameters as implied by the results of [13]. We thank Diego Blas for the discussion of this point.

to be spatially flat we substitute the homogeneous isotropic Ansatz,

$$ds^2 = N(t)^2 dt^2 - a(t)^2 \mathbf{dx}^2, \quad (12a)$$

$$u_0 = N(t), \quad u_i = 0, \quad (12b)$$

$$\Theta = \Theta(t), \quad (12c)$$

into the action,

$$S_{[EH]} + S_{[u]} + S_{[\Theta]} = \int d^4x a^3 \left[ -\frac{M_0^2(6 + 3\beta + 9\lambda)}{2} \frac{\dot{a}^2}{Na^2} + \frac{\dot{\Theta}^2}{2Nc_\Theta^2} - \mu^2 \dot{\Theta} - VN \right], \quad (13)$$

where we have introduced the notation

$$c_\Theta^2 = \frac{1}{1 + \varkappa}. \quad (14)$$

Varying this expression with respect to  $\Theta$  and  $N$  we obtain,

$$\frac{d}{dt} \left[ a^3 \left( \frac{\dot{\Theta}}{c_\Theta^2} - \mu^2 \right) \right] = 0, \quad (15)$$

$$H^2 = \frac{1}{3M_0^2(1 + \beta/2 + 3\lambda/2)} \left( \frac{\dot{\Theta}^2}{2c_\Theta^2} + V \right), \quad (16)$$

where we fixed the gauge  $N = 1$  and introduced the Hubble parameter  $H \equiv \dot{a}/a$ . From Eq. (15) we see that the time derivative of the inflaton develops a non zero VEV in the stationary regime,

$$\dot{\Theta} = \mu^2 c_\Theta^2. \quad (17)$$

This is precisely the type of the background relevant for the ghost inflation. Note that the kinetic energy of  $\Theta$  contributes into the r.h.s. of the Friedmann equation (16), which makes accelerated expansion possible even in the absence of any potential,  $V = 0$  [30].

For inflation to end, the shift symmetry (10) must be broken. We will incorporate this by promoting the parameters in (11) to slowly varying functions<sup>7</sup> of  $\Theta$ ,

$$\varkappa \mapsto \varkappa(\Theta), \quad \mu \mapsto \mu(\Theta), \quad V \mapsto V(\Theta). \quad (18)$$

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<sup>7</sup>As noted above, the parameters of the aether / khronon Lagrangian (2) can, in principle, also change during or after inflation. Including their dependence on the inflaton  $\Theta$  is straightforward but renders the formulas rather cumbersome. For the sake of clarity we will assume that they stay constant during the period of inflation responsible for the generation of the observed part of the perturbation spectrum.

This does not affect the Friedmann equation (16), while the equation for  $\Theta$  gets modified. Instead of (15) we obtain,

$$\ddot{\Theta} + 3H(\dot{\Theta} - \mu^2 c_\Theta^2) + c_\Theta^2 V' - \frac{c'_\Theta}{c_\Theta} \dot{\Theta}^2 = 0, \quad (19)$$

where prime stands for the derivative with respect to  $\Theta$ . One can now identify two different inflationary regimes.

**1. Fast-roll inflation.** It corresponds to the case when the contribution of the additional terms related to the breaking of the shift symmetry is small, so that the time derivative of the inflaton is still approximately given by (17). To find the conditions for the validity of this regime, we write

$$\dot{\Theta} = \mu^2 c_\Theta^2 (1 + \delta^2) \quad (20)$$

with

$$\delta^2 \ll 1 \quad (21)$$

and substitute it into (19). Neglecting the terms with time derivatives of  $\delta^2$  (the validity of this approximation will be discussed shortly) we obtain,

$$\delta^2 = -\frac{1}{3H} \left( 2\mu\mu' c_\Theta^2 + \mu^2 c_\Theta c'_\Theta + \frac{V'}{\mu^2} \right). \quad (22)$$

The condition (21) is satisfied provided that

$$\frac{\mu'}{\mu}, \frac{c'_\Theta}{c_\Theta} \ll \frac{H}{\mu^2 c_\Theta^2}, \quad (23a)$$

$$V' \ll \mu^2 H. \quad (23b)$$

These inequalities have clear physical interpretation. The conditions (23a) can be written as  $\frac{\dot{\mu}}{\mu}, \frac{\dot{c}_\Theta}{c_\Theta} \ll H$ , i.e. the variations of the parameters during the Hubble time are small. The third inequality (23b) states that the contribution of the potential into the force acting on the inflaton is small compared to the contribution which arises from the coupling to the aether.

Finally, in deriving (22) we have neglected the terms in (19) with time-derivatives of  $\delta^2$ . It is straightforward to see that this is justified as long as all quantities in (22) change slowly, as one expects during a quasi-de Sitter stage. In addition to (23) this implies,

$$\frac{\mu''}{\mu'}, \frac{c''_\Theta}{c'_\Theta}, \frac{V''}{V'} \lesssim \frac{H}{\mu^2 c_\Theta^2}. \quad (24)$$



An interesting limit of the fast-roll inflation arises when the potential represents only a small correction to the kinetic energy or is absent altogether,

$$\mu^4 \gg V. \quad (25)$$

Then the Hubble rate is directly related to the VEV of the inflaton time derivative (or ghost condensate in the language of ghost inflation),

$$H^2 \approx \frac{\mu^4 c_\Theta^2}{6M_0^2}. \quad (26)$$

We will call this limit **kinetically driven inflation** and will study it in detail in Sec. 5.

**2. Slow-roll inflation.** This occurs when the inequality (23b) is replaced by the reverse,

$$V' \gg \mu^2 H. \quad (27)$$

In this case the background dynamics of the inflaton is essentially the same as in the standard potentially-driven inflation. This regime has been recently analyzed in detail in [29] where it was concluded that the aether-inflaton coupling either leads to tachyonic instability of the cosmological perturbations or its effect is unobservably small. In this paper we concentrate on the opposite regime of the fast-roll where the behavior of perturbations is quite different as we are going to see now.

### 3 Linear perturbations

We start the analysis of the perturbations by briefly discussing the tensor modes. These are not modified by the introduction of the inflaton  $\Theta$  and behave exactly in the same way as in pure Einstein-aether or khronometric theory. Thus we can use the known result [32] that the only modification of the primordial gravity wave power spectrum is the change of normalization due to deviation of the velocity of tensor modes  $c_t$  from unity,<sup>8</sup>

$$\mathcal{P}_h(k) = \frac{2}{\pi^2 c_t} \frac{H^2}{M_0^2} \Big|_{c_t k = aH}, \quad \text{where} \quad c_t^2 = \frac{1}{1 - \beta} \quad (28)$$

and  $k$  is the comoving wavenumber. This is a small effect under our assumption  $\beta \ll 1$ .

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<sup>8</sup>The factor 2 compared to Eq. (29) of [32] comes from the sum over two polarizations of the gravity waves.

The analysis of other excitations is greatly simplified by the observation that the dynamics of the aether (khronon) and inflaton decouple from those of gravity in the regime of interest. To show this we write

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} , \quad u_\mu = \bar{u}_\mu + v_\mu , \quad \Theta = \bar{\Theta} + \pi , \quad (29)$$

where the background quantities  $\bar{g}_{\mu\nu}$ ,  $\bar{u}_\mu$ ,  $\bar{\Theta}$  are given by Eqs. (12) with  $N(t) = 1$ . We want to compare the terms mixing  $h_{\mu\nu}$  with  $v_\mu$  and  $\pi$  in the quadratic Lagrangian to the kinetic terms for these fluctuations. Let us first show that the mixing between  $h_{\mu\nu}$  and  $v_\mu$  can be neglected provided<sup>9</sup>  $\alpha \ll 1$ . Schematically one has the following estimates

$$\mathcal{L}_{[EH]}^{(2)} + \mathcal{L}_{[u]}^{(2)} \sim \frac{M_0^2}{2} (\partial h)^2 + \frac{M_0^2 \alpha}{2} [(\partial v)^2 + \partial v \partial h + (\partial h)^2] \sim \frac{(\partial \hat{h})^2}{2} + \frac{(\partial \hat{v})^2}{2} + \sqrt{\frac{\alpha}{1+\alpha}} \frac{\partial \hat{v} \partial \hat{h}}{2} , \quad (30)$$

where in the second expression we have introduced the canonically normalized fields  $\hat{v}_\mu$ ,  $\hat{h}_{\mu\nu}$ . We see that, indeed, the mixing term is suppressed by  $\sqrt{\alpha}$ . The analysis of the inflaton–metric mixing requires more work as it involves non-trivial cancellations between various terms. It is performed in Appendix A where it is demonstrated that in the leading approximation the mixing can be neglected. This means that we can consistently set the metric to its unperturbed value and consider only excitations of the aether/khronon and inflaton in the external quasi-de Sitter background. In Appendix B we check explicitly that the corrections to this approximation are small by computing the quadratic effective action for scalar inflationary perturbations in the uniform inflaton gauge with fully dynamical metric.

The quadratic action for the perturbations reads,

$$S_{[u]}^{(2)} + S_{[\Theta]}^{(2)} = \int d^4x a^3 \left[ M_0^2 (c_1 + c_4) \frac{\dot{v}_i^2}{2a^2} - M_0^2 c_1 \frac{\partial_i v_k \partial_i v_k}{2a^4} - M_0^2 (c_2 + c_3) \frac{(\partial_i v_i)^2}{2a^4} \right. \\ \left. + \frac{(1 + \varkappa) \dot{\pi}^2}{2} - \frac{(\partial_i \pi)^2}{2a^2} + (\mu^2 - \varkappa \dot{\Theta}) \frac{v_i \partial_i \pi}{a^2} - (\mu^2 - \varkappa \dot{\Theta}) \dot{\Theta} \frac{v_i^2}{2a^2} \right] . \quad (31)$$

In deriving this expression we have neglected several types of contributions. First, we neglected the terms coming from the Taylor expansion of the functions  $\varkappa(\Theta)$ ,  $\mu(\Theta)$  and  $V(\Theta)$ . These terms give an effective mass for the field  $\pi$  which, due to the conditions (23), (24) is much smaller than the Hubble rate. Thus they are irrelevant for the calculation of the primordial power spectrum determined by the dynamics of the perturbations inside the Hubble radius and at horizon crossing. Second, we discarded terms of the form

$$O(\alpha) M_0^2 H^2 v_i^2 / a^2$$

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<sup>9</sup>Recall that  $\alpha$  collectively denotes the order of magnitude of all coefficients  $c_a$  in the aether Lagrangian.

appearing in the aether Lagrangian. These are smaller than the last term in (31) provided

$$H^2 \ll \frac{\mu^4}{M_0^2 \alpha} . \quad (32)$$

For the kinetically driven inflation this condition is automatically fulfilled. We will assume it to be valid also in the presence of the potential, as only in this case we obtain the interesting phenomenology of the ghost inflation.

Following the standard procedure one decomposes  $v_i$  into the transverse and longitudinal parts,

$$v_i = v_i^{(t)} + \partial_i \chi , \quad \partial_i v_i^{(t)} = 0 . \quad (33)$$

Note that the transverse part is present only in the case of the general aether, while it identically vanishes in the khronometric case. In the former case the equation for it follows from (31),

$$\ddot{v}_i^{(t)} + H \dot{v}_i^{(t)} - \frac{c_1}{c_1 + c_4} \frac{\Delta v_i^{(t)}}{a^2} + \frac{\mu^4 c_\Theta^4}{M_0^2 (c_1 + c_4)} v_i^{(t)} = 0 , \quad (34)$$

where  $\Delta \equiv \partial_i \partial_i$ . We observe that the last term gives a mass to the transverse vector modes which, according to (32), is much larger than the Hubble rate (we assume throughout the paper that  $c_\Theta$  is of order 1). This implies that these modes are not generated during inflation which justifies our previous assertion. Thus for the purposes of this paper the general aether is equivalent to its khronometric reduction.

Henceforth we concentrate on the scalar sector of perturbations. Note that in the khronometric model  $\chi$  coincides with the perturbation of the khronon field which we write as,<sup>10</sup>

$$\sigma = t + \chi . \quad (35)$$

The equations for the coupled system of the fields  $\chi$  and  $\pi$  are,

$$\ddot{\chi} + H \dot{\chi} - c_\chi^2 \frac{\Delta \chi}{a^2} + \frac{\mu^2 - \varkappa \dot{\Theta}}{M_0^2 \alpha} (\dot{\Theta} \chi - \pi) = 0 , \quad (36a)$$

$$\ddot{\pi} + 3H \dot{\pi} - c_\Theta^2 \frac{\Delta \pi}{a^2} + (\mu^2 - \varkappa \dot{\Theta}) c_\Theta^2 \frac{\Delta \chi}{a^2} = 0 , \quad (36b)$$

where we have introduced the notation

$$c_\chi^2 = \frac{\beta + \lambda}{\alpha} . \quad (37)$$

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<sup>10</sup>The background value of the khronon can always be brought to the form  $\bar{\sigma} = t$  using the reparameterization (7).

Let us study the subhorizon behavior of the modes. Neglecting the terms containing  $H$  and performing the Fourier decomposition,  $\chi, \pi \propto e^{-i\omega t + i\mathbf{k}\mathbf{x}}$ , we obtain the dispersion relations,

$$\omega_{\pm}^2 = \frac{c_{\chi}^2 + c_{\Theta}^2}{2} \left[ \frac{k^2}{a^2} + k_c^2 \pm \sqrt{\left( \frac{k^2}{a^2} + k_c^2 \right)^2 - \frac{4c_{\Theta}^2 c_{\chi}^2}{(c_{\chi}^2 + c_{\Theta}^2)^2} \frac{k^4}{a^4} - \frac{4\delta^2 k_c^2}{c_{\chi}^2 + c_{\Theta}^2} \frac{k^2}{a^2}} \right], \quad (38)$$

where

$$k_c^2 \equiv \frac{\mu^4 c_{\Theta}^4}{M_0^2 (c_{\chi}^2 + c_{\Theta}^2) \alpha}. \quad (39)$$

Note that (32) implies  $k_c \gg H$ . At large momenta,  $k/a \gg k_c$ , the fields  $\chi$  and  $\pi$  decouple and the dispersion relations become linear,

$$\omega_+^2 = c_{\chi}^2 (k/a)^2, \quad \omega_-^2 = c_{\Theta}^2 (k/a)^2, \quad (40)$$

where the velocities  $c_{\chi}, c_{\Theta}$  in general differ from unity as the consequence of the Lorentz violation. At small momenta,  $k/a \ll k_c$ , on the contrary, the mixing is large and the dispersion relations take the form,

$$\omega_+^2 = (c_{\chi}^2 + c_{\Theta}^2) k_c^2 + (c_{\chi}^2 + c_{\Theta}^2 - 2\delta^2) (k/a)^2, \quad (41)$$

$$\omega_-^2 = \delta^2 (k/a)^2 + \frac{c_{\chi}^2 c_{\Theta}^2}{c_{\chi}^2 + c_{\Theta}^2} \frac{(k/a)^4}{k_c^2}, \quad (42)$$

where  $\delta$  is given by Eq. (22). From (41) we see that one family of perturbations acquires an energy gap much larger than the Hubble rate. Thus it is irrelevant for the inflationary physics and can be integrated out. Effectively we are dealing with a single field inflation<sup>11</sup> described by the second mode which remains gapless, see (42). Remarkably, the linear term in its dispersion relation is strongly suppressed by the small parameter  $\delta^2$ ; in the limit of the exact shift symmetry of the inflaton the linear contribution disappears altogether and one is left with the quadratic dispersion relation  $\omega^2 \propto k^4$ . This exactly coincides with the situation in the tilted ghost inflation [18, 19]. Note, however, that our setup violates an assumption often present in the discussion of the ghost inflation that the scale suppressing the  $k^4$  term in the dispersion relation is of the same order as the scale of the ghost condensate. In our case we have from (20), (39),

$$k_c \sim \frac{\dot{\Theta}}{M_0 \sqrt{\alpha}}. \quad (43)$$

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<sup>11</sup>This implies absence of isocurvature perturbations.

The validity of the EFT description for the background requires  $\sqrt{\dot{\bar{\Theta}}}$  to be smaller than the cutoff of the theory (8), which implies the hierarchy

$$k_c \ll \sqrt{\dot{\bar{\Theta}}} . \quad (44)$$

We emphasize that this hierarchy naturally appears in the framework which captures both the background evolution and perturbations within a single consistent EFT.

To find the low-energy effective action for the gapless perturbations one notes that at  $k/a \ll k_c$  equation (36a) is dominated by the last term which tightly couples  $\chi$  to  $\pi$ ,

$$\chi = \pi / \dot{\bar{\Theta}} . \quad (45)$$

Substituting this back into (31) yields the quadratic action,

$$S_{[\pi]}^{(2)} = \int d^4x \frac{a^3}{2c_{\bar{\Theta}}^2} \left[ \dot{\pi}^2 - \delta^2 \frac{(\partial_i \pi)^2}{a^2} - \frac{c_{\chi}^2 c_{\bar{\Theta}}^2}{c_{\chi}^2 + c_{\bar{\Theta}}^2} \frac{(\Delta \pi)^2}{k_c^2 a^4} \right] , \quad (46)$$

where we have neglected the terms containing spatial derivatives of  $\dot{\pi}$  as they are irrelevant at low momenta. Clearly, this action reproduces the dispersion relation (42). The subsequent analysis proceeds differently depending on whether the dispersion relation is dominated by the first or second term in (42) when the mode freezes out, which occurs when the frequency of the mode drops down to the Hubble rate,  $\omega \sim H$ .

### Quadratic dispersion relation

By inspection of Eq. (42) one finds that the first term on the r.h.s. is never important if

$$\delta^2 \ll H/k_c . \quad (47)$$

Then the mode equation following from (46) has the form,

$$\ddot{\pi}_{\mathbf{k}} + 3H\dot{\pi}_{\mathbf{k}} + \frac{c_{\chi}^2 c_{\bar{\Theta}}^2}{c_{\chi}^2 + c_{\bar{\Theta}}^2} \frac{k^4}{k_c^2 a^4} \pi_{\mathbf{k}} = 0 . \quad (48)$$

One quantizes the  $\pi$ -field,

$$\pi(\mathbf{x}, t) = \int \frac{d^3k}{(2\pi)^3} (\pi_{\mathbf{k}}(t) a_{\mathbf{k}} + \pi_{\mathbf{k}}^*(t) a_{-\mathbf{k}}^+) e^{i\mathbf{k}\mathbf{x}} , \quad (49)$$

with the creation-annihilation operators obeying the standard commutation relations

$$[a_{\mathbf{k}}, a_{\mathbf{q}}^+] = (2\pi)^3 \delta^3(\mathbf{k} - \mathbf{q}) .$$

The mode functions  $\pi_{\mathbf{k}}(t)$  are the positive-frequency solutions of Eq. (48),

$$\pi_{\mathbf{k}} = \sqrt{\frac{\pi}{8}} c_{\Theta} H |\eta|^{3/2} H_{3/4}^{(1)} \left( \frac{c_{\chi} c_{\Theta} H}{2(c_{\chi}^2 + c_{\Theta}^2)^{1/2} k_c} k^2 \eta^2 \right), \quad (50)$$

where  $\eta = \int dt/a$  is the conformal time and  $H_{3/4}^{(1)}$  is the Hankel function. The normalization of (50) is fixed by imposing the canonical commutation relations on  $\pi(\mathbf{x}, t)$  and its conjugate momentum following from (46). The power spectrum of  $\pi$  is given by the formula

$$\mathcal{P}_{\pi} = \frac{k^3}{2\pi^2} \lim_{\eta \rightarrow 0} |\pi_{\mathbf{k}}(\eta)|^2. \quad (51)$$

We are interested in the power spectrum of the gauge invariant perturbation  $\zeta$  which stays constant outside the horizon [34, 35]. This is defined as the fluctuation of the logarithm of the scale factor on the surfaces of constant inflaton field. In our approximation of the decoupling between the inflaton and the metric perturbations and neglecting small deviations of the background space-time from de Sitter  $\zeta$  is related to  $\pi$  by a simple formula<sup>12</sup> (cf. [33]),

$$\zeta = -\frac{H}{\dot{\Theta}} \pi. \quad (52)$$

Combining everything together and using the Taylor expansion for the Hankel functions we obtain,

$$\mathcal{P}_{\zeta}(k) = \frac{1}{\pi(\Gamma(1/4))^2} \cdot \frac{1}{c_{\chi}^{3/2} c_{\Theta}^{1/2}} \cdot \frac{H^{5/2}}{\mu M_0^{3/2} \alpha^{3/4}} \Big|_{a^2 H = \frac{c_{\chi} M_0 \sqrt{\alpha}}{c_{\Theta} \mu^2} k^2}, \quad (53)$$

where we have emphasized explicitly that all the quantities entering into this expression must be evaluated at the moment when the mode freezes out. The slow evolution of these quantities gives a tilt of the power spectrum,

$$n_s - 1 \equiv \frac{d \log \mathcal{P}_{\zeta}}{d \log k} = -\frac{\mu^2 c_{\Theta}^2}{2H} \left( \frac{c'_{\Theta}}{c_{\Theta}} + \frac{2\mu'}{\mu} \right) - \frac{5\mu^4 c_{\Theta}^2}{4M_0^2 H^2} \delta^2. \quad (54)$$

Finally, the tensor-to-scalar ratio is obtained using (28),

$$r \equiv \frac{\mathcal{P}_t}{\mathcal{P}_{\zeta}} = \frac{2(\Gamma(1/4))^2}{\pi} \cdot \frac{c_{\chi}^{3/2} c_{\Theta}^{1/2} \mu}{M_0^{1/2} H^{1/2}} \cdot \alpha^{3/4}, \quad (55)$$

where we have neglected the difference of the graviton speed from 1. In Sec. 5 we will use these expressions to make numerical estimates in the special case of the kinetically-driven inflation.

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<sup>12</sup>For the analysis in the next section it is important to stress that within the above approximation this relation holds at non-linear order [18].

## Linear dispersion relation

If

$$\delta^2 \gg H/k_c \quad (56)$$

the dispersion relation of the modes (42) is dominated by the linear term at the moment of freeze-out. This means that we can neglect the last term in the action (46) when deriving the mode equation,

$$\ddot{\pi} + 3H\dot{\pi} + \frac{k^2\delta^2}{a^2}\pi = 0. \quad (57)$$

Its normalized positive frequency solution is

$$\pi_{\mathbf{k}} = \frac{Hc_{\Theta}}{\sqrt{2k\delta}}|\eta|\left(1 - \frac{i}{k\eta\delta}\right)e^{-ik\eta\delta}. \quad (58)$$

This gives the power spectrum,

$$\mathcal{P}_{\zeta}(k) = \frac{1}{4\pi^2} \cdot \frac{H^4}{\mu^4 c_{\Theta}^2 \delta^3} \Big|_{aH=k\delta}, \quad (59)$$

the tilt,

$$n_s - 1 = -\frac{2\mu^2 c_{\Theta}^2}{H} \left( \frac{c'_{\Theta}}{c_{\Theta}} + \frac{2\mu'}{\mu} \right) - \frac{2\mu^4 c_{\Theta}^2}{M_0^2 H^2} \delta^2 - \frac{3\dot{\delta}}{H\delta} \quad (60)$$

and the tensor-to-scalar ratio,

$$r = \frac{8\mu^4 c_{\Theta}^2}{M_0^2 H^2} \cdot \delta^3. \quad (61)$$

## 4 Bispectrum

The interesting feature of ghost inflation is enhanced non-Gaussianity of a special shape. In this section we are going to show that this feature is reproduced in our model by computing the three-point function of the curvature perturbations.

As a first step we must identify the leading interaction terms. To this end we expand the aether and inflaton Lagrangians to the cubic order in the perturbations. This task is simplified by several observations. First, due to the decoupling of gravity, the metric perturbations are not excited and one can set them to zero. Second, the vector modes of the aether are not excited either as they are massive during inflation. Thus the aether is reduced to its longitudinal — khronon — part. Third, for an estimate we drop terms with time derivatives of the scale factor arising in the expansion of the aether action; this is a valid

approximation<sup>13</sup> for subhorizon modes with  $\omega \gtrsim H$ . Finally, we neglect contributions coming from the expansion of the functions  $\varkappa(\Theta)$ ,  $\mu(\Theta)$ ,  $V(\Theta)$  as, by assumption, the deviation of the space-time from de Sitter is small and these contributions are suppressed. In this way one obtains,<sup>14</sup>

$$\mathcal{L}_{[u]}^{(3)} = M_0^2 \alpha \left( \frac{\dot{\chi} \partial_i \ddot{\chi} \partial_i \chi}{a^2} - \frac{\partial_i \dot{\chi} \partial_j \chi \partial_i \partial_j \chi}{a^4} \right) + \frac{M_0^2 (\beta + \lambda)}{a^4} (2 \partial_i \dot{\chi} \partial_i \chi \Delta \chi + \dot{\chi} (\Delta \chi)^2) , \quad (62)$$

$$\mathcal{L}_{[\Theta]}^{(3)} = (\mu^2 - \varkappa \dot{\Theta}) \dot{\Theta} \frac{\dot{\chi} (\partial_i \chi)^2}{a^2} - \left( \frac{\mu^2}{2} - \varkappa \dot{\Theta} \right) \frac{(\partial_i \chi)^2 \dot{\pi}}{a^2} - (\mu^2 - \varkappa \dot{\Theta}) \frac{\dot{\chi} \partial_i \chi \partial_i \pi}{a^2} - \varkappa \frac{\partial_i \chi \dot{\pi} \partial_i \pi}{a^2} . \quad (63)$$

We want to compare various terms in (62), (63) at the time when the perturbations freeze out. The frequency and momentum of the mode then satisfy

$$H \sim \omega \ll k/a \ll k_c , \quad (64)$$

where the last two inequalities follow from the condition (32) and the dispersion relation (42). This implies that the spatial gradients are enhanced relative to the time derivatives. Using also the relation (45) between  $\chi$  and  $\pi$  valid at low frequency we obtain the estimates,

$$\mathcal{L}_{[u]}^{(3)} \sim \frac{M_0^2 \alpha}{\mu^6} \omega (k/a)^4 \pi^3 , \quad \mathcal{L}_{[\Theta]}^{(3)} \sim \frac{1}{\mu^2} \omega (k/a)^2 \pi^3 . \quad (65)$$

From (64) one immediately concludes  $\mathcal{L}_{[u]}^{(3)} \ll \mathcal{L}_{[\Theta]}^{(3)}$ . In other words, the cubic interaction is dominated by the contribution from the inflaton Lagrangian. Expressing  $\chi$  through  $\pi$  and substituting the background value  $\dot{\Theta}$  one ends up with

$$S_{[\pi]}^{(3)} = - \int d^4 x \frac{a}{2 \mu^2 c_\Theta^4} \dot{\pi} (\partial_i \pi)^2 . \quad (66)$$

This has precisely the same form as in the ghost inflation model [18].

The rest of the analysis of the bispectrum proceeds along the lines of [18, 19]. To get an order-of-magnitude estimate for the size of non-linear effects consider the ratio between the cubic and quadratic Lagrangians at the moment of freeze-out,

$$\frac{\mathcal{L}_{[\pi]}^{(3)}}{\mathcal{L}_{[\pi]}^{(2)}} \sim \frac{\omega (k/a)^2 \pi^3 / \mu^2}{\omega^2 \pi^2} \sim \frac{(k/a)^2}{\mu^2 H} \pi . \quad (67)$$

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<sup>13</sup>For the inflaton action we do not need this simplification as no derivatives of the scale factor appear in it. Thus Eq. (63) is valid also for superhorizon modes.

<sup>14</sup>To simplify (62) we have integrated by parts in the action. Note that the term with higher time derivatives in (62) can be removed by the field redefinition [3]  $\chi = \bar{\chi} + \bar{\chi} \dot{\chi}$  and thus does not lead to any new degrees of freedom.



The amplitude of the bispectrum is characterized by the quantity  $f_{NL}$  which is related to (67) by [33],

$$|f_{NL}| \sim \frac{1}{\zeta} \frac{\mathcal{L}_{[\pi]}^{(3)}}{\mathcal{L}_{[\pi]}^{(2)}} \sim \frac{(k/a)^2}{H^2} \Big|_{\text{freeze-out}}, \quad (68)$$

where we have used the relation (52) between  $\zeta$  and  $\pi$ . Depending on whether the dispersion relation of the perturbations is dominated by the quadratic or linear term, we obtain,

$$|f_{NL}| \sim \begin{cases} k_c/H & \text{if } \delta^2 \ll H/k_c \\ 1/\delta^2 & \text{if } \delta^2 \gg H/k_c \end{cases} \quad (69)$$

In both cases the non-Gaussianity is parametrically enhanced,  $|f_{NL}| \gg 1$ . We point out, however, that it is smaller than in the original ghost inflation model. To see this let us rewrite (69) using the expressions for the power spectrum (53), (59). Dropping the numerical coefficients we obtain,

$$|f_{NL}| \sim \begin{cases} \zeta^{-4/5} (k_c/\mu)^{8/5} & \text{if } \delta^2 \ll \zeta^{4/5} (\mu/k_c)^{8/5} \\ 1/\delta^2 & \text{if } \delta^2 \gg \zeta^{4/5} (\mu/k_c)^{8/5} \end{cases} \quad (70)$$

The predictions of the original ghost inflation are recovered in the limit  $k_c \sim \mu$ . The hierarchy of scales (44) clearly introduces a suppression.

The precise expression for the bispectrum is obtained by evaluating the three-point function of the inflaton fluctuation  $\pi$  at the moment of freeze-out  $t_f$  and then switching to  $\zeta$  using (52). Using the in-in perturbation theory [36] with the interaction Lagrangian (66) one obtains for the correlator of three Fourier harmonics,

$$\begin{aligned} \langle \pi(\mathbf{k}_1, t_f) \pi(\mathbf{k}_2, t_f) \pi(\mathbf{k}_3, t_f) \rangle &= i \int_{-\infty}^{t_f} dt d^3x \frac{-a(t)}{2\mu^2 c_\Theta^4} \\ &\quad \times \langle [\pi(\mathbf{k}_1, t_f) \pi(\mathbf{k}_2, t_f) \pi(\mathbf{k}_3, t_f), \dot{\pi}(\mathbf{x}, t) (\partial_i \pi(\mathbf{x}, t))^2] \rangle_0, \end{aligned} \quad (71)$$

where on the r.h.s. the expectation value is evaluated in the unperturbed vacuum. We are interested in the function  $B_\zeta$  defined as,

$$\langle \zeta(\mathbf{k}_1) \zeta(\mathbf{k}_2) \zeta(\mathbf{k}_3) \rangle = (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B_\zeta(k_1, k_2, k_3), \quad (72)$$

where  $\delta$ -function appears due to the translation invariance while rotation symmetry implies that  $B_\zeta$  depends only on the absolute values of the momenta. Substituting the decomposition

(49) into (71) one obtains after a bit of algebra,

$$B_\zeta(k_1, k_2, k_3) = i \frac{H^2}{\mu^8 c_\Theta^{10}} (\mathbf{k}_2 \cdot \mathbf{k}_3) \lim_{\eta_f \rightarrow 0^-} \pi_{\mathbf{k}_1}(\eta_f) \pi_{\mathbf{k}_2}(\eta_f) \pi_{\mathbf{k}_3}(\eta_f) \\ \times \text{Re} \int_{-\infty}^{\eta_f} \frac{d\eta}{\eta} \frac{d\pi_{\mathbf{k}_1}^*(\eta)}{d\eta} \pi_{\mathbf{k}_2}^*(\eta) \pi_{\mathbf{k}_3}^*(\eta) + \text{perm.} , \quad (73)$$

where we have switched to integration over conformal time and “perm.” stands for the terms differing by permutations of the momenta. The final answer depends on the precise form of the modes at the freeze-out.

The case of **quadratic dispersion relation** yields,

$$B_\zeta(k_1, k_2, k_3) = -\frac{H^4}{M_0^4 c_\chi^4 \alpha^2} \cdot \frac{\pi^3}{16(\Gamma(1/4))^3} \frac{k_1(k_1^2 - k_2^2 - k_3^2)}{(k_1 k_2 k_3)^3} I\left(\frac{k_2}{k_1}, \frac{k_3}{k_1}\right) + \text{perm.} , \quad (74)$$

where

$$I(x_2, x_3) = \text{Re} \int_{-(1+i\epsilon)\infty}^0 \frac{dy}{y} \frac{dF(y)}{dy} F(x_2 y) F(x_3 y) \quad (75)$$

and

$$F(y) = (-y)^{3/2} H_{3/4}^{(1)}(y^2/2) . \quad (76)$$

Note that in (75) we have rotated the contour of integration below the real axis to make the integral convergent. The shape of the bispectrum obtained by numerically computing (75) is shown in Fig. 1 (left panel) as the function of two ratios,  $x_{2,3} = k_{2,3}/k_1$  [37]. Clearly, it is maximal for equilateral configurations  $k_1 \sim k_2 \sim k_3$ .

To quantify its amplitude one evaluates the bispectrum at equal momenta and introduces the parameter<sup>15</sup>  $f_{NL}$ ,

$$B_\zeta(k, k, k) = \frac{3}{5} \cdot 6 \cdot \left( \frac{2\pi^2}{k^3} \mathcal{P}_\zeta(k) \right)^2 f_{NL} . \quad (77)$$

Comparison with (74), (53) yields,

$$f_{NL} = \frac{5\pi\Gamma(1/4)}{192} I(1, 1) \frac{\mu^2 c_\Theta}{H M_0 c_\chi \sqrt{\alpha}} \approx -0.13 \frac{\mu^2 c_\Theta}{H M_0 c_\chi \sqrt{\alpha}} . \quad (78)$$

Note that  $f_{NL}$  is predicted to be negative. By the absolute value (78) agrees with the estimate (69). Notice though a numerical factor giving suppression by about an order of magnitude.

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<sup>15</sup>This definition of  $f_{NL}$  agrees with the conventions used by the Planck collaboration [20] upon substitution  $\Phi = 3\zeta/5$ ,  $P_\Phi = 2\pi^2 \mathcal{P}_\Phi/k^3$ .

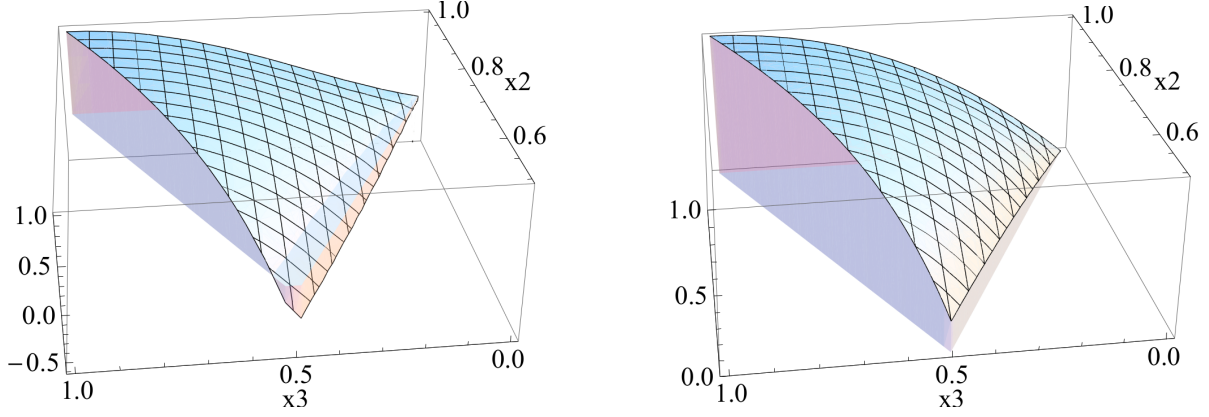


Figure 1: The shapes of the bispectrum  $B_\zeta(1, x_2, x_3)x_2^2x_3^2$  for the cases of quadratic (left panel), and linear dispersion relations (right panel). The functions have been normalized to one at  $x_2 = x_3 = 1$  and are plotted only for inequivalent configurations of momenta  $1 - x_2 \leq x_3 \leq x_2$ .

In the case of **linear dispersion relation** we obtain,

$$B_\zeta(k_1, k_2, k_3) = \frac{H^8}{\mu^8 c_\Theta^4 \delta^8} \cdot \frac{1}{16} \frac{k_1^2(k_1^2 - k_2^2 - k_3^2)}{(k_1 k_2 k_3)^3} \frac{k_1^2 + 2k_2^2 + 2k_3^2 + 3k_1 k_2 + 3k_1 k_3 + 6k_2 k_3}{(k_1 + k_2 + k_3)^3} + \text{perm.} \quad (79)$$

The momentum dependence can be cast into the form,

$$B_\zeta(k_1, k_2, k_3) \propto \frac{\sum_a k_a^6 + \sum_{a \neq b} (3k_a^5 k_b - k_a^4 k_b^2 - 3k_a^3 k_b^3) + \sum_{a \neq b \neq c} (3k_a^4 k_b k_c - 9k_a^3 k_b^2 k_c - 2k_a^2 k_b^2 k_c^2)}{(k_1 k_2 k_3)^3 (k_1 + k_2 + k_3)^3}, \quad (80)$$

which is one of the templates used by the Planck collaboration in the data analysis, see [20] where it is called “EFT1”. The shape (79) is plotted on the right panel of Fig. 1. It is similar to the previous case, but there are visible differences. From Eqs. (79), (59) one infers,

$$f_{NL} = -\frac{85}{324} \cdot \frac{1}{\delta^2} \approx -\frac{0.26}{\delta^2}. \quad (81)$$

Again, we see that  $f_{NL}$  is negative and agrees with the estimate (69). Remarkably, in this case it depends on the single parameter  $\delta^2$ .

## 5 Kinetically driven inflation

If the potential for the  $\Theta$ -field is absent,  $V = 0$ , the Hubble rate is given by the expression (26) which simplifies the formulas and makes the model highly predictive. The condition

(47) for the dominance of the quadratic piece in the dispersion relation translates into

$$\delta^2 \ll \sqrt{\alpha} \quad (82)$$

and we obtain for the main observables,

$$\left. \begin{array}{l} \text{quadratic} \\ \text{dispersion} \end{array} \right\} \begin{cases} \mathcal{P}_\zeta = 2.6 \cdot 10^{-3} \frac{c_\Theta^2}{c_\chi^{3/2}} \frac{\mu^4}{M_0^4 \alpha^{3/4}} , & (83a) \\ n_s - 1 = -6 \delta^2 , & (83b) \\ r = 13 c_\chi^{3/2} \alpha^{3/4} , & (83c) \\ f_{NL} = -\frac{0.32}{c_\chi \sqrt{\alpha}} . & (83d) \end{cases}$$

In the opposite case

$$\delta^2 \gg \sqrt{\alpha} \quad (84)$$

corresponding to the linear dispersion relation we have

$$\left. \begin{array}{l} \text{linear} \\ \text{dispersion} \end{array} \right\} \begin{cases} \mathcal{P}_\zeta = 0.7 \cdot 10^{-3} \frac{\mu^4 c_\Theta^2}{M_0^4} \frac{1}{\delta^3} , & (85a) \\ n_s - 1 = -6 \delta^2 - \frac{3\dot{\delta}}{H\delta} , & (85b) \\ r = 48 \delta^3 , & (85c) \\ f_{NL} = -\frac{0.26}{\delta^2} . & (85d) \end{cases}$$

These expressions should be compared with the best-fit values for the cosmological parameters reported by the Planck collaboration [38, 39],

$$\ln(10^{10} \mathcal{P}_\zeta) \Big|_{k=0.05 \text{Mpc}^{-1}} = 3.089^{+0.024}_{-0.027} \quad (68\% \text{ CL}) , \quad (86a)$$

$$n_s = 0.9603 \pm 0.0073 \quad (68\% \text{ CL}) , \quad (86b)$$

$$r < 0.11 \quad (95\% \text{ CL}) \quad (86c)$$

and the constraints on the two relevant forms of non-Gaussianity — the “ghost inflation” shape and the “EFT1” shape in the notations of [20],

$$f_{NL}^{\text{ghost}} = -23 \pm 88 \quad (68\% \text{ CL}) , \quad (87a)$$

$$f_{NL}^{\text{EFT1}} = 8 \pm 73 \quad (68\% \text{ CL}) . \quad (87b)$$

We first analyze the case of **quadratic dispersion relation**. From (83b), (86b) one finds,

$$\delta^2 \approx 6.6 \cdot 10^{-3} . \quad (88)$$

Then Eq. (82) gives us the condition for the validity of the regime at hand,

$$\alpha \gg 4.4 \cdot 10^{-5} . \quad (89)$$

On the other hand, an upper bound on  $\alpha$  follows from the constraint on the tensor-to-scalar ratio (86c),

$$\alpha < 1.7 \cdot 10^{-3} c_\chi^{-2} . \quad (90)$$

Assuming  $c_\chi \sim 1$  we conclude that the model with quadratic dispersion relation is allowed in a rather narrow parameter range around  $\alpha \sim 10^{-4}$ . For the level of non-Gaussianity one obtains,

$$f_{NL} = -\frac{32}{c_\chi} \left( \frac{10^{-4}}{\alpha} \right)^{1/2} , \quad (91)$$

comfortably within the allowed region (87a). Finally, from (83a), (86a) we determine the energy scale of inflation,

$$\mu \approx 5.4 \cdot 10^{-3} \frac{c_\chi^{3/8}}{c_\Theta^{1/2}} \left( \frac{\alpha}{10^{-4}} \right)^{3/16} M_0 . \quad (92)$$

This is rather high, unlike the original model of ghost inflation [18], which explains the tendency of our model towards large tensor-to-scalar ratio.

When

$$\alpha \ll 4.4 \cdot 10^{-5} \quad (93)$$

we are in the regime of **linear dispersion relation**. Now the spectral index receives an additional contribution related to the time variation of  $\delta$ , so that  $\delta^2$  cannot be precisely determined any longer. Still, without fine-tuning (85b) implies

$$\delta^2 \lesssim 6.6 \cdot 10^{-3} . \quad (94)$$

Then

$$r = 0.026 \left( \frac{\delta^2}{6.6 \cdot 10^{-3}} \right)^{3/2} \quad (95)$$

is below the Planck sensitivity but may be accessible to future missions [40]. The predicted non-Gaussianity is

$$f_{NL} = -39 \left( \frac{6.6 \cdot 10^{-3}}{\delta^2} \right) , \quad (96)$$

again within the Planck constraint (87b). For the scale of inflation we obtain,

$$\mu \approx \frac{6.4 \cdot 10^{-3}}{c_\Theta^{1/2}} \left( \frac{\delta^2}{6.6 \cdot 10^{-3}} \right)^{3/8} M_0 . \quad (97)$$

Last but not least, we have to check that the scale of inflation is below the theory cutoff (8), so that the model stays within the validity of effective field theory. By substituting the numbers in (92) we find that for the case of quadratic dispersion relation this requirement is (marginally) fulfilled. On the other hand, for the linear dispersion relation the validity of the EFT requires

$$\frac{4 \cdot 10^{-5}}{c_\Theta} \left( \frac{\delta^2}{6.6 \cdot 10^{-3}} \right)^{3/4} \lesssim \alpha, \quad (98)$$

which is at tension with (93). Thus we are forced to conclude that in the kinetically driven limit the regime of purely linear dispersion dominance is incompatible with the EFT description. Rather, at  $\alpha \sim 5 \cdot 10^{-5}$  both terms in the dispersion relation (42) are important and the whole equation following from (46) must be solved to find the evolution of the fluctuation modes. This requires numerical integration, which is beyond the scope of the present paper. One expects that the results for the observables will interpolate between the formulas (83) and (85). The shape of the bispectrum will be a mixture of the two shapes plotted on Fig. 1.

It is worth emphasizing that the problem with the inflation scale hitting the cutoff appears only in the kinetically driven limit and is easily avoided by adding the inflaton potential.

## 6 Conclusions

In this paper we presented a setup that provides a UV-completion of the well-known ghost inflation model up to a scale which can be almost as high as the Planck mass. This is achieved by coupling the inflaton to the Lorentz-violating sector described by the Einstein-aether theory or its khronometric version. The cutoff of our model coincides with the cutoff of the Einstein-aether sector and is unrelated to the scale of the “ghost condensate” — the dynamically developed expectation value for the time derivative of the inflaton. In the khronometric version the construction can be potentially completed even further in the framework of the Hořava gravity. Curiously, the UV-completion occurs without restoration of the broken Lorentz symmetry, cf. [23].

We have studied the inflationary evolution of the Universe and the generation of primordial perturbations in the model. The latter are described by an effective theory containing a single degree of freedom and governed by the same effective action as in the ghost inflation. The novelty of our model compared to the previous works on ghost inflation is that it allows to go beyond the study of small perturbations and incorporates in a unified framework the background dynamics. We have found that the ghost condensate gives positive contribution

into the energy budget of the Universe. This makes inflation possible even in the absence of any potential for the inflaton — the regime that we called kinetically driven inflation. In principle, it is straightforward to incorporate in our model the graceful exit from inflation and reheating which proceeds through vanishing<sup>16</sup> of the inflaton potential and the ghost condensate. This would be impossible in the original formulation of the ghost inflation as the low-energy EFT because its cutoff goes to zero in this limit.

On the phenomenological side, we have calculated the characteristics of the power spectrum and bispectrum predicted by the model. Specifically, in the kinetically driven regime the model predicts a rather high tensor-to-scalar ratio and is already constrained by the bounds from the Planck mission. The non-Gaussianity is predicted to be close to the equilateral type with the amplitude  $f_{NL} \sim -40$ . This is still well within the limits set by Planck. Optimistically, one can expect it to be probed by planned surveys [40, 41, 42].

There are two directions in which our work can be extended. First, additional insight will be gained from the study of higher statistics. In the general EFT formulation of ghost inflation the trispectrum depends on an additional free coupling constant standing in front of the quartic interaction of inflaton perturbations  $(\partial_i \pi)^2 (\partial_j \pi)^2$  [43]. On the other hand, in our model this coupling is unambiguously determined by the specific form of the UV-completed theory. Repeating the analysis presented in the beginning of Sec. 4 it is straightforward to find the leading quartic interaction

$$S_{[\pi]}^{(4)} = \int d^4x \frac{1}{8\mu^4 c_\Theta^6 a} (\partial_i \pi)^2 (\partial_j \pi)^2.$$

Clearly, the quartic coupling is related the cubic one, see Eq. (66), and thus the shape of the trispectrum will be uniquely predicted.

Second, it is interesting to investigate if the UV-completion along the lines presented in this paper can be found for other Lorentz-violating models of modified gravity. From the theory viewpoint, the case of Lorentz-violating massive gravity [44] deserves particular attention as its completion above the scale of strong coupling associated to the graviton mass could be considered as the analog of the Higgs mechanism in massive Yang–Mills theory.

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<sup>16</sup>At least up to the tiny present value of the dark energy density. We do not consider a fine-tuned situation when after inflation the ghost condensate stays large and its positive energy is cancelled by a negative cosmological constant.

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## A Inflaton–metric mixing

In this Appendix we show that for  $\alpha \ll 1$  the mixing between the inflaton and the metric can be neglected in the analysis of the primordial perturbations. Besides the terms included in (31), the  $\Theta$ -action (11) contains the following terms mixing the inflaton and the metric,<sup>17</sup>

$$\mathcal{L}_{h\pi} = -\frac{(1+\varkappa)\dot{\Theta}}{2} h^{00}\dot{\pi} + ((1+\varkappa)\dot{\Theta} - \mu^2) \left( \frac{1}{2} h_i^i \dot{\pi} - h^{0i} \partial_i \pi \right), \quad (99)$$

where the indices on the metric components have been raised using the background metric  $\bar{g}^{\mu\nu}$ . For the sake of the argument, in deriving (99) we neglected the dependence of the functions  $\varkappa$ ,  $\mu$ ,  $V$  on  $\Theta$ . We see that the mixing term contains only one derivative as opposed to the two-derivative kinetic terms for  $h_{\mu\nu}$  and  $\pi$  and thus is irrelevant above certain frequency  $\omega_{mix}$  and momentum  $k_{mix}$  [33]. We need to verify that  $\omega_{mix}$  is smaller than the Hubble parameter  $H$ , so that the mixing stays negligible when the mode freezes out.

As explained in the main text, the behavior of the  $\pi$ -modes is qualitatively different depending on whether the physical momentum  $k/a$  is larger or smaller than  $k_c$  given by (39). For  $k/a > k_c$  the frequency of the mode is of the same order as the momentum,  $\omega \sim k/a$  and the mixing is estimated as,

$$\mathcal{L}_{[h\pi]} \sim -\frac{\mu^2}{2} h \partial \pi - \mu^2 \delta^2 h \partial \pi, \quad (100)$$

where we have used the background value (20) for  $\dot{\Theta}$ . Taking into account that  $\omega > k_c$  we see that the first term is smaller than the kinetic terms for  $h_{\mu\nu}$  and  $\pi$  by a factor  $\sqrt{\alpha}$ , while the second term is further suppressed by  $\delta^2$ .

At  $k/a < k_c$  the dispersion relation for  $\pi$  is given by (42) and implies  $\omega \ll k$ . Then in the equation of motion for the metric perturbations we can neglect the terms with time

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<sup>17</sup>We note that  $S_{[\Theta]}$  does not produce any mixing between  $h_{\mu\nu}$  and  $v_\mu$  in addition to that present in  $S_{[u]}$ .



derivatives. This yields schematically,

$$M_0^2 k^2 h \sim \mu^2 \dot{\pi} \implies \mathcal{L}_{[h\pi]} \sim \frac{\mu^4}{k^2 M_0^2} \dot{\pi}^2, \quad (101)$$

where we have omitted the contribution with the spatial derivative of  $\pi$  as it is always subdominant. For the modes at freeze-out,  $\omega \sim H$ ,

$$\mathcal{L}_{[h\pi]} \sim \delta^2 \frac{\mu^4}{M_0^2 H^2} \dot{\pi}^2 \quad \text{or} \quad \sqrt{\alpha} \frac{\mu^2}{M_0 H} \dot{\pi}^2, \quad (102)$$

depending on whether the dispersion relation (42) is dominated by the first or the second term. The Friedmann equation (16) implies<sup>18</sup>  $H \gtrsim \mu^2/M_0$ . Thus the contribution (102) is in both cases parametrically suppressed compared to the first term in the  $\pi$ -Lagrangian (46). We conclude that the mixing between  $\pi$  and the metric can be consistently neglected.

## B Scalar perturbations: the uniform inflaton gauge

To verify the frozen-metric approximation adopted in the main text we calculate the effective quadratic action for scalar perturbations allowing the metric to be dynamical. For concreteness, we concentrate on the khronometric version of the theory and work in the gauge comoving with the khronon, where  $\sigma = t$ . As we saw in Sec. 3, at low momenta the perturbations of the inflaton are tied to those of the khronon, see Eq. (45), implying that surfaces of constant khronon are also surfaces of constant inflaton. Thus in the chosen gauge the inflaton is uniform on the constant-time slices,  $\Theta = \bar{\Theta}(t)$ .

Writing the metric in the ADM form,

$$ds^2 = N^2 dt^2 - \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \quad (103)$$

we obtain the action

$$S \equiv S_{[EH]} + S_{[w]} + S_{[\Theta]} = \int d^4x N \sqrt{\gamma} \left[ \frac{M_0^2}{2} \left( (1 - \beta) K_{ij} K^{ij} - (1 + \lambda) K^2 + \mathcal{R} + \alpha \frac{(\partial_i N)^2}{N^2} \right) + \frac{(1 + \varkappa) \dot{\Theta}^2}{N^2} - \frac{\mu^2 \dot{\Theta}}{N} - V \right], \quad (104)$$

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<sup>18</sup>We exclude the fine-tuned situation when the kinetic and potential energy of the inflaton nearly cancel each other.

where

$$K_{ij} = \frac{\dot{\gamma}_{ij} - \nabla_i N_j - \nabla_j N_i}{2N}$$

is the extrinsic curvature of constant-time slices,  $K$  is its trace,  $\mathcal{R}$  is the intrinsic curvature constructed from the 3d-metric  $\gamma_{ij}$ ; the indices are raised and lowered using  $\gamma_{ij}$  and its inverse. Next, we specify to the sector of scalar perturbations,

$$N = 1 + \phi, \quad N^i = \partial_i \psi, \quad \gamma_{ij} = a^2 e^{2\zeta} \delta_{ij},$$

and expand the action up to quadratic order,

$$S^{(2)} = \int d^4x \left[ -a^3 V \phi^2 + a M_0^2 \alpha \frac{(\partial_i \phi)^2}{2} + 6a^2 \dot{a} M_c^2 \phi \dot{\zeta} - 2a M_0^2 \phi \Delta \zeta - 2\dot{a} M_c^2 \phi \Delta \psi \right. \\ \left. - M_0^2 (\lambda + \beta) \frac{(\Delta \psi)^2}{2a} + 2a M_c^2 \dot{\zeta} \Delta \psi - 3a^3 M_c^2 \dot{\zeta}^2 + a M_0^2 (\partial_i \zeta)^2 \right], \quad (105)$$

where  $M_c^2 = M_0^2(1 + \beta/2 + 3\lambda/2)$ . In deriving this expression we made use of the background equations of motion (16), (19). Variation of (105) with respect to  $\phi$ ,  $\psi$  produces the constraints,

$$-2a^3 V \phi - a M_0^2 \alpha \Delta \phi + 6a^2 \dot{a} M_c^2 \dot{\zeta} - 2a M_0^2 \Delta \zeta - 2\dot{a} M_c^2 \Delta \psi = 0, \quad (106a)$$

$$-M_0^2 (\lambda + \beta) \frac{\Delta \psi}{a} - 2\dot{a} M_c^2 \phi + 2a M_c^2 \dot{\zeta} = 0. \quad (106b)$$

Solving these equations with respect to  $\phi$  and  $\psi$ , substituting the result back into (105) and retaining only terms of order up to  $O(\alpha)$  one obtains,

$$S^{(2)} = \int d^4x \frac{a^3(1 + \varkappa)}{2} \frac{\dot{\Theta}^2}{H^2} \left[ \left( 1 - \frac{(\lambda + \beta)(1 + \varkappa)\dot{\Theta}^2}{4M_0^2 H^2} \right) \dot{\zeta}^2 - \tilde{\delta}^2 \frac{(\partial_i \zeta)^2}{a^2} - \frac{(\lambda + \beta)M_0^2}{(1 + \varkappa)\dot{\Theta}^2} \frac{(\Delta \zeta)^2}{a^4} \right] \quad (107)$$

with

$$\tilde{\delta}^2 = -\frac{2M_0^2 \dot{H}}{(1 + \varkappa)\dot{\Theta}} - \frac{\lambda + \beta}{2} = \delta^2 - \frac{\lambda + \beta}{2}, \quad (108)$$

where  $\delta$  is defined in (22). Recalling the expression for the inflaton background (20) one observes that upon the substitution (52) the action coincides with the formula (46) obtained in the main text up to negligible corrections in the time-derivative term and the replacement  $\delta^2 \mapsto \tilde{\delta}^2$ . Let us show that the latter difference is never important. Indeed, if  $\delta^2 \lesssim \lambda + \beta$ , then  $\delta^2$  also satisfies the inequality (47) and the dispersion relation is dominated by the quadratic contribution. This implies that the second term in (107) can be neglected altogether. On the other hand, if  $\delta^2$  satisfies (56) we have  $\delta^2 \gg H/k_c \sim \sqrt{\alpha} \gg \lambda + \beta$ , and the difference between  $\tilde{\delta}^2$  and  $\delta^2$  is subleading.

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